

No-Arbitrage Condition of Option Implied Volatility and Bandwidth Selection

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ABSTRACT A standard approach to option pricing is based on Black-Scholes type (BS hereafter) models utilizing the no-arbitrage argument of complete markets. However, there are several crucial assumptions, such as that the option underlying log-returns follow normal distribution, there is unique and deterministic riskless rate as well as the volatility of underlying log-returns. Since the assumptions are generally not fulfilled, the BS-type models mostly provide false results. A common market practice is therefore to invert option pricing model and using market prices of highly liquid options to get a so called implied volatility (IV). The BS model at one time moment can be related to the whole set of IVs as given by maturity/moneyness relation of tradable options. One can therefore get IV curve or surface (a so called smirk or smile). Since the moneyness and maturity of IV often do not match the data of valuated options, some sort of estimating and local smoothing is necessary. However, it can lead to arbitrage opportunity, if no-arbitrage conditions on state price density (SPD) are ignored. In this paper, using option data on DAX index, we analyze the behavior of IV and SPD with respect to different choices of bandwidth parameter h and a set of bandwidths, which violates no-arbitrage conditions, are identified. Moreover, it is documented that the change of h implies interesting changes in the violation interval of moneyness. We also show the impact of h on the total area of SPD under zero, which can be seen as a degree of no-arbitrage violation.